Counting

Finite Math

25 January 2019

Basic Counting

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Addition Principle

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Now, suppose that every freshmen who is majoring in Chemistry is enrolled in Calculus or in History. If there are 20 freshmen Chemistry majors enrolled in Calculus and 15 freshmen Chemistry majors enrolled in History. How many total freshmen Chemistry majors are there?

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Theorem (Addition Principle for Counting)

For any two sets A and B,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

Now You Try It!

Example

According to a survey of business firms in a certain city, 345 *firms offer their employees group life insurance,* 285 *offer long-term disability insurance, and* 115 *offer group life insurance and long-term disability insurance. How many firms offer their employees group life insurance or long-term disability insurance?*

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According to a survey of business firms in a certain city, 345 *firms offer their employees group life insurance,* 285 *offer long-term disability insurance, and* 115 *offer group life insurance and long-term disability insurance. How many firms offer their employees group life insurance or long-term disability insurance?*

Solution

515

Multiplication Principle

Example

Suppose a store has 3 *types of shirts, and in each type of shirt, they have* 4 *colors available. How many options are available?*

Multiplication Principle

Theorem (Multiplication Principle for Counting)

1. *If two operations O_1 and O_2 are performed in order, with N_1 possible outcomes for the first operation and N_2 possible outcomes for the second operation, then there are*

$$N_1 \cdot N_2$$

possible combined outcomes of the first operation followed by the second operation.

2. *In general, if n operations $O_1, O_2, ..., O_n$ are performed in order, with possible number of number of outcomes $N_1, N_2, ..., N_n$, respectively, then there are*

$$N_1 \cdot N_2 \cdots N_n$$

possible combined outcomes of the operations performed in the given order.

Now You Try It!

Example

Suppose a 6-sided die and a 12-sided die are rolled. How many different possible outcomes are there?

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Suppose a 6-sided die and a 12-sided die are rolled. How many different possible outcomes are there?

Solution

72

More Multiplication Principle

Example

Suppose we have a list of 8 letters that we wish to make code words from. How many possible 4-letter code words can be made if:

(a) *letters can be repeated?*

(b) *no letter can be repeated?*

(c) *adjacent letters cannot be alike?*

Now You Try It!

Example

Suppose we have a list of 10 letters that we wish to make code words from. How many possible 5-letter code words can be made if:

(a) *letters can be repeated?*

(b) *no letter can be repeated?*

(c) *adjacent letters cannot be alike?*

Now You Try It!

Example

Suppose we have a list of 10 *letters that we wish to make code words from. How many possible* 5*-letter code words can be made if:*

(a) *letters can be repeated?*

(b) *no letter can be repeated?*

(c) *adjacent letters cannot be alike?*

Solution

(a) 100, 000*, (b)* 30, 240*, (c)* 65, 610

Combining Rules

Example

There are 30 teams in the MLB. Suppose a store sells both fitted and snapback baseball caps. Suppose the store carries standard and alternate versions of the fitted cap for each team, but only the standard version of the cap for the snapback cap. How many total different baseball caps do they sell?